

Полярные координаты

Задан закон движения точки в полярных координатах: $\rho = \rho(t)$ (в метрах), $\varphi = \varphi(t)$. В указанный момент времени найти скорость и ускорение точки в полярных, декартовых и естественных координатах.

Кирсанов М.Н. **Решебник. Теоретическая механика**/Под ред. А. И. Кириллова.– М.:ФИЗМАТЛИТ, 2002.– 384 с. (с.144.)

Задача 4.1.

6

$$r = 45(1 - (t/9)^2)/t,$$
$$\varphi = \arccos(t/9), \quad t = 4 \text{ с.}$$

Задача 4.2.

6

$$r = \frac{16 \sin^2(t/11)}{\cos(t/11)},$$
$$\varphi = \frac{t}{11}, \quad t = 8 \text{ с.}$$

Задача 4.3.

6

$$r = 26/(1 + t/45),$$
$$\varphi = \arccos(t/9), \quad t = 8 \text{ с.}$$

Задача 4.4.

6

$$r = 5(t/14 + 0.5)^{-5},$$
$$\varphi = (t/14 + 0.5)^5, \quad t = 7 \text{ с.}$$

Задача 4.5.

6

$$r = \frac{27}{1 + \cos(t/11)},$$
$$\varphi = \frac{t}{11}, \quad t = 10 \text{ с.}$$

Задача 4.6.

6

$$r = 11e^{t/6},$$
$$\varphi = e^{t/6}, \quad t = 4 \text{ с.}$$

Задача 4.7.

6

$$r = 24/(1 + 2t/25),$$
$$\varphi = \arccos(t/10), \quad t = 6 \text{ с.}$$

Задача 4.8.

6

$$r = \frac{30}{1 + \cos(t/13)},$$
$$\varphi = \frac{t}{13}, \quad t = 10 \text{ с.}$$

Задача 4.9.

6

$$r = \frac{24}{1 + 5 \cos(t/8)},$$
$$\varphi = \frac{t}{8}, \quad t = 7 \text{ с.}$$

Задача 4.10.

6

$$r = 11 \cos(t/3) + 12,$$
$$\varphi = t/3, \quad t = 1 \text{ с.}$$

Задача 4.11.

6

$$r = 15(t/10)^4,$$
$$\varphi = (t/10)^4, \quad t = 9 \text{ с.}$$

Задача 4.12.

6

$$r = 3t/2 + 6,$$
$$\varphi = \arccos(t/6), \quad t = 5 \text{ с.}$$

Задача 4.13.

6

$$r = \frac{26}{1 + \cos(t/13)},$$
$$\varphi = \frac{t}{13}, t = 10 \text{ c.}$$

Задача 4.14.

6

$$r = 25 / (1 + \frac{2}{9}t),$$
$$\varphi = \arccos(t/9), t = 8 \text{ c.}$$

Задача 4.15.

6

$$r = 22 \cos^2(\pi t/13),$$
$$\varphi = \cos^2(\pi t/13), t = 8 \text{ c.}$$

Задача 4.16.

6

$$r = 11(t/20 + 0.5)^{-3},$$
$$\varphi = (t/20 + 0.5)^3, t = 10 \text{ c.}$$

Задача 4.17.

6

$$r = \frac{25}{1 + 0.2 \cos(t/11)},$$
$$\varphi = \frac{t}{11}, t = 10 \text{ c.}$$

Задача 4.18.

6

$$r = \frac{24}{1 + \cos(t/5)},$$
$$\varphi = \frac{t}{5}, t = 4 \text{ c.}$$

Задача 4.19.

6

$$r = \frac{23}{1 + \cos(t/4)},$$
$$\varphi = \frac{t}{4}, t = 1 \text{ c.}$$

Задача 4.20.

6

$$r = 2t \cos(t/5),$$
$$\varphi = t, t = 4 \text{ c.}$$

Задача 4.21.

6

$$r = \frac{22}{1 + \cos(t/10)},$$
$$\varphi = \frac{t}{10}, t = 9 \text{ c.}$$

Задача 4.22.

6

$$r = 36(1 - (t/4)^2)/t,$$
$$\varphi = \arccos(t/4), t = 2 \text{ c.}$$

Задача 4.23.

6

$$r = 4t - \frac{32}{t},$$
$$\varphi = \arccos(t/4), t = 2 \text{ c.}$$

Задача 4.24.

6

$$r = 11e^{t/24},$$
$$\varphi = t/4, t = 2 \text{ c.}$$

Задача 4.25.

6

$$r = \frac{22}{1 + 3 \cos(t/2)},$$
$$\varphi = \frac{t}{2}, t = 1 \text{ c.}$$

Задача 4.26.

6

$$r = \frac{28}{1 + 2 \cos(t/8)},$$
$$\varphi = \frac{t}{8}, t = 7 \text{ c.}$$

Задача 4.27.

6

$$r = \frac{30}{1 + \cos(t/8)},$$
$$\varphi = \frac{t}{8}, t = 7 \text{ c.}$$

Задача 4.28.

6

$$r = 12 \cos^2(\pi t/7),$$
$$\varphi = \cos^2(\pi t/7), t = 4 \text{ c.}$$

Задача 4.29.

6

$$r = 21(t/10)^4,$$
$$\varphi = (t/10)^4, t = 9 \text{ c.}$$

Задача 4.30.

6

$$r = 8t \cos(t/7),$$
$$\varphi = t, t = 6 \text{ c.}$$

Задача 4.31.

6

$$r = 8e^{-t/8},$$
$$\varphi = e^{t/8}, t = 4 \text{ c.}$$

Задача 4.32.

6

$$r = 10 + 10t \operatorname{tg}^2(\pi t/11),$$
$$\varphi = \cos^2(\pi t/11), t = 3 \text{ c.}$$

Задача 4.33.

6

$$r = 36(1 - (t/9)^2)/t,$$
$$\varphi = \arccos(t/9), t = 7 \text{ c.}$$

Задача 4.34.

6

$$r = -\frac{13 \cos(t/6)}{\cos(t/12)},$$
$$\varphi = \frac{t}{12}, t = 10 \text{ c.}$$

Полярные координаты

№	ρ	$\dot{\rho}$	φ	$\dot{\varphi}$	v_ρ	v_φ	v	v_x	v_y	Кривая
1	9.028	-3.368	1.110	-0.124	-3.368	-1.120	3.549	-0.494	-3.515	Циссоида
2	9.467	2.700	0.727	0.091	2.700	0.861	2.834	1.445	2.438	Циссоида
3	22.075	-0.417	0.476	-0.243	-0.417	-5.354	5.370	2.083	-4.950	Эллипс
4	5.000	-1.786	1.000	0.357	-1.786	1.786	2.525	-2.467	-0.538	Гиперболическая спираль
5	16.724	0.743	0.909	0.091	0.743	1.520	1.692	-0.743	1.520	Парабола
6	21.425	3.571	1.948	0.325	3.571	6.955	7.818	-7.781	0.760	Архимедова спираль
7	16.216	-0.877	0.927	-0.125	-0.877	-2.027	2.208	1.096	-1.917	Эллипс
8	17.458	0.544	0.769	0.077	0.544	1.343	1.449	-0.544	1.343	Парабола
9	5.708	0.651	0.875	0.125	0.651	0.713	0.966	-0.130	0.957	Гипербола
10	22.395	-1.200	0.333	0.333	-1.200	7.465	7.561	-3.576	6.661	Улитка Паскаля
11	9.841	4.374	0.656	0.292	4.374	2.870	5.231	1.715	4.942	Архимедова спираль
12	13.500	1.500	0.586	-0.302	1.500	-4.070	4.338	3.500	-2.563	Улитка Паскаля
13	15.130	0.471	0.769	0.077	0.471	1.164	1.256	-0.471	1.164	Парабола
14	9.000	-0.720	0.476	-0.243	-0.720	-2.183	2.299	0.360	-2.270	Гипербола
15	2.766	3.526	0.126	0.160	3.526	0.443	3.553	3.442	0.882	Архимедова спираль
16	11.000	-1.650	1.000	0.150	-1.650	1.650	2.333	-2.280	-0.497	Гиперболическая спираль
17	22.264	0.284	0.909	0.091	0.284	2.024	2.044	-1.422	1.468	Эллипс
18	14.145	1.196	0.800	0.200	1.196	2.829	3.071	-1.196	2.829	Парабола
19	11.682	0.367	0.250	0.250	0.367	2.920	2.943	-0.367	2.920	Парабола
20	5.574	0.246	4.000	1.000	0.246	5.574	5.579	4.058	-3.829	
21	13.567	0.655	0.900	0.100	0.655	1.357	1.507	-0.655	1.357	Парабола
22	13.500	-11.250	1.047	-0.289	-11.250	-3.897	11.906	-2.250	-11.691	Циссоида
23	-8.000	12.000	1.047	-0.289	12.000	2.309	12.220	4.000	11.547	Строфоида
24	11.956	0.498	0.500	0.250	0.498	2.989	3.030	-0.996	2.862	Логарифмическая спираль
25	6.056	1.199	0.500	0.500	1.199	3.028	3.257	-0.400	3.232	Гипербола
26	12.270	1.032	0.875	0.125	1.032	1.534	1.848	-0.516	1.775	Гипербола
27	18.282	1.069	0.875	0.125	1.069	2.285	2.523	-1.069	2.285	Парабола
28	0.594	2.337	0.050	0.195	2.337	0.116	2.340	2.328	0.231	Архимедова спираль
29	13.778	6.124	0.656	0.292	6.124	4.018	7.324	2.401	6.919	Архимедова спираль
30	31.421	0.053	6.000	1.000	0.053	31.421	31.421	8.830	30.155	
31	4.852	-0.607	1.649	0.206	-0.607	1.000	1.170	-0.950	-0.683	Гиперболическая спираль
32	23.319	15.372	0.429	-0.283	15.372	-6.592	16.725	16.721	0.397	Гиперболическая спираль
33	2.032	-1.179	0.680	-0.177	-1.179	-0.359	1.233	-0.691	-1.020	Циссоида
34	1.851	3.377	0.833	0.083	3.377	0.154	3.381	2.157	2.603	Строфоида

№	$\ddot{\rho}$	$\ddot{\varphi}$	W_ρ	W_φ	a	W_x	W_y	$ W_\tau $	W_n
1	1.406	-0.008	1.267	0.767	1.481	-0.123	1.476	-1.444	0.328
2	0.556	0.000	0.478	0.491	0.685	0.031	0.685	0.605	0.323
3	0.016	-0.114	-1.283	-2.318	2.649	-0.079	-2.648	2.410	1.099
4	0.765	0.102	0.128	-0.765	0.776	0.713	-0.306	-0.631	0.451
5	0.119	0.000	-0.020	0.135	0.136	-0.119	0.068	0.113	0.077
6	0.595	0.054	-1.663	3.478	3.855	-2.621	-2.826	2.334	3.067
7	0.095	-0.012	-0.159	0.029	0.161	-0.118	-0.109	0.036	0.157
8	0.077	0.000	-0.026	0.084	0.088	-0.077	0.042	0.068	0.056
9	0.217	0.000	0.127	0.163	0.207	-0.043	0.202	0.206	0.016
10	-1.155	0.000	-3.643	-0.800	3.730	-3.181	-1.948	-0.212	3.724
11	1.458	0.097	0.621	3.508	3.562	-1.647	3.158	2.443	2.592
12	-0.000	-0.137	-1.227	-2.755	3.016	0.500	-2.974	2.160	2.104
13	0.067	0.000	-0.023	0.072	0.076	-0.067	0.036	0.059	0.048
14	0.115	-0.114	-0.414	-0.678	0.794	-0.058	-0.792	0.774	0.181
15	1.923	0.087	1.852	1.372	2.305	1.666	1.593	2.009	1.130
16	0.330	0.015	0.083	-0.330	0.340	0.322	-0.109	-0.292	0.175
17	0.027	0.000	-0.157	0.052	0.165	-0.137	-0.092	0.029	0.162
18	0.435	0.000	-0.131	0.478	0.496	-0.435	0.239	0.390	0.307
19	0.382	0.000	-0.348	0.183	0.393	-0.382	0.092	0.139	0.368
20	-0.797	0.000	-6.370	0.491	6.389	4.536	4.500	0.210	6.386
21	0.115	0.000	-0.020	0.131	0.133	-0.115	0.066	0.109	0.075
22	9.000	-0.048	7.875	5.846	9.808	-1.125	9.743	-9.355	2.946
23	-8.000	-0.048	-7.333	-6.543	9.828	2.000	-9.623	-8.438	5.040
24	0.021	0.000	-0.726	0.249	0.768	-0.757	-0.130	0.126	0.758
25	1.572	0.000	0.058	1.199	1.200	-0.524	1.080	1.136	0.388
26	0.281	0.000	0.089	0.258	0.273	-0.141	0.234	0.264	0.070
27	0.237	0.000	-0.049	0.267	0.272	-0.237	0.134	0.221	0.158
28	4.355	0.363	4.333	1.126	4.477	4.272	1.339	4.383	0.910
29	2.041	0.097	0.870	4.911	4.987	-2.306	4.421	3.421	3.629
30	-2.369	0.000	-33.790	0.106	33.790	-32.415	9.543	0.049	33.790
31	0.076	0.026	-0.130	-0.125	0.181	0.135	-0.120	-0.039	0.176
32	19.003	0.023	17.140	-8.149	18.979	18.977	-0.284	18.964	0.734
33	0.210	-0.039	0.146	0.338	0.369	-0.099	0.355	-0.239	0.281
34	0.581	0.000	0.568	0.563	0.800	-0.035	0.799	0.593	0.536