

Полярные координаты

Задан закон движения точки в полярных координатах: $\rho = \rho(t)$ (в метрах), $\varphi = \varphi(t)$. В указанный момент времени найти скорость и ускорение точки в полярных, декартовых и естественных координатах.

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Задача 4.1.

7

$$r = \frac{26}{1 + \cos(t/13)},$$
$$\varphi = \frac{t}{13}, t = 10 \text{ с.}$$

Задача 4.2.

7

$$r = \frac{29}{1 + 0.4 \cos(t/6)},$$
$$\varphi = \frac{t}{6}, t = 4 \text{ с.}$$

Задача 4.3.

7

$$r = \frac{27}{1 + 4 \cos(t/5)},$$
$$\varphi = \frac{t}{5}, t = 4 \text{ с.}$$

Задача 4.4.

7

$$r = \frac{6}{7}t - \frac{21}{t},$$
$$\varphi = \arccos(t/7), t = 4 \text{ с.}$$

Задача 4.5.

7

$$r = \frac{4}{5}t - \frac{10}{t},$$
$$\varphi = \arccos(t/5), t = 1 \text{ с.}$$

Задача 4.6.

7

$$r = 24/(1 + t/7),$$
$$\varphi = \arccos(t/7), t = 3 \text{ с.}$$

Задача 4.7.

7

$$r = 16 \cos^2(\pi t/14),$$
$$\varphi = \cos^2(\pi t/14), t = 10 \text{ с.}$$

Задача 4.8.

7

$$r = 35(1 - (t/5)^2)/t,$$
$$\varphi = \arccos(t/5), t = 1 \text{ с.}$$

Задача 4.9.

7

$$r = 4(t/16 + 0.5)^{-3},$$
$$\varphi = (t/16 + 0.5)^3, t = 8 \text{ с.}$$

Задача 4.10.

7

$$r = 5(t/16 + 0.5)^{-2},$$
$$\varphi = (t/16 + 0.5)^2, t = 8 \text{ с.}$$

Задача 4.11.

7

$$r = 3 + 3 \operatorname{tg}^2(\pi t/25),$$
$$\varphi = \cos^2(\pi t/25), t = 8 \text{ с.}$$

Задача 4.12.

7

$$r = 7t \sin(t/7),$$
$$\varphi = t, t = 6 \text{ с.}$$

Задача 4.13.

7

$$r = 20 \cos^2(\pi t/14),$$
$$\varphi = \cos^2(\pi t/14), \quad t = 9 \text{ с.}$$

Задача 4.14.

7

$$r = 5t - \frac{40}{t},$$
$$\varphi = \arccos(t/4), \quad t = 2 \text{ с.}$$

Задача 4.15.

7

$$r = 30(1 - (t/5)^2)/t,$$
$$\varphi = \arccos(t/5), \quad t = 2 \text{ с.}$$

Задача 4.16.

7

$$r = \frac{12 \sin^2(t/8)}{\cos(t/8)},$$
$$\varphi = \frac{t}{8}, \quad t = 4 \text{ с.}$$

Задача 4.17.

7

$$r = 2 \cos(t/4) + 2,$$
$$\varphi = t/4, \quad t = 3 \text{ с.}$$

Задача 4.18.

7

$$r = 11e^{-t/7},$$
$$\varphi = e^{t/7}, \quad t = 5 \text{ с.}$$

Задача 4.19.

7

$$r = 10e^{t/10},$$
$$\varphi = e^{t/10}, \quad t = 8 \text{ с.}$$

Задача 4.20.

7

$$r = 27/(1 + \frac{4}{3}t),$$
$$\varphi = \arccos(t/3), \quad t = 2 \text{ с.}$$

Задача 4.21.

7

$$r = 22/(1 + \frac{2}{7}t),$$
$$\varphi = \arccos(t/7), \quad t = 6 \text{ с.}$$

Задача 4.22.

7

$$r = 11 + 11 \operatorname{tg}^2(\pi t/23),$$
$$\varphi = \cos^2(\pi t/23), \quad t = 7 \text{ с.}$$

Задача 4.23.

7

$$r = 27(1 - (t/9)^2)/t,$$
$$\varphi = \arccos(t/9), \quad t = 5 \text{ с.}$$

Задача 4.24.

7

$$r = 19e^{-t/10},$$
$$\varphi = e^{t/10}, \quad t = 6 \text{ с.}$$

Задача 4.25.

7

$$r = 21(1 - (t/7)^2)/t,$$
$$\varphi = \arccos(t/7), \quad t = 4 \text{ с.}$$

Задача 4.26.

7

$$r = \frac{4}{9}t - \frac{18}{t},$$
$$\varphi = \arccos(t/9), \quad t = 7 \text{ с.}$$

Задача 4.27.

7

$$r = 28/(1 + t/5),$$
$$\varphi = \arccos(t/5), \quad t = 2 \text{ c.}$$

Задача 4.28.

7

$$r = 9t/4 + 4,$$
$$\varphi = \arccos(t/4), \quad t = 2 \text{ c.}$$

Задача 4.29.

7

$$r = 5 \cos(t/7) + 5,$$
$$\varphi = t/7, \quad t = 6 \text{ c.}$$

Задача 4.30.

7

$$r = 100/t + 10,$$
$$\varphi = \arccos(t/10), \quad t = 9 \text{ c.}$$

Задача 4.31.

7

$$r = 24/(1 + 4t/45),$$
$$\varphi = \arccos(t/9), \quad t = 5 \text{ c.}$$

Задача 4.32.

7

$$r = \frac{28}{1+4 \cos(t/7)},$$
$$\varphi = \frac{t}{7}, \quad t = 6 \text{ c.}$$

Задача 4.33.

7

$$r = -\frac{16 \cos(2t/11)}{\cos(t/11)},$$
$$\varphi = \frac{t}{11}, \quad t = 6 \text{ c.}$$

Задача 4.34.

7

$$r = 81/t + 9,$$
$$\varphi = \arccos(t/9), \quad t = 8 \text{ c.}$$

Полярные координаты

№	ρ	$\dot{\rho}$	φ	$\dot{\varphi}$	v_ρ	v_φ	v	v_x	v_y	Кривая
1	15.130	0.471	0.769	0.077	0.471	1.164	1.256	-0.471	1.164	Парабола
2	22.064	0.692	0.667	0.167	0.692	3.677	3.742	-1.730	3.318	Эллипс
3	7.130	1.081	0.800	0.200	1.081	1.426	1.789	-0.270	1.769	Гипербола
4	-1.821	2.170	0.963	-0.174	2.170	0.317	2.193	0.980	1.962	Строфоида
5	-9.200	10.800	1.369	-0.204	10.800	1.878	10.962	0.320	10.957	Строфоида
6	16.800	-1.680	1.128	-0.158	-1.680	-2.656	3.143	1.680	-2.656	Парабола
7	6.220	3.500	0.389	0.219	3.500	1.361	3.756	2.723	2.586	Архимедова спираль
8	33.600	-36.400	1.369	-0.204	-36.400	-6.859	37.041	-0.560	-37.036	Циссоида
9	4.000	-0.750	1.000	0.188	-0.750	0.750	1.061	-1.036	-0.226	Гиперболическая спираль
10	5.000	-0.625	1.000	0.125	-0.625	0.625	0.884	-0.864	-0.188	Гиперболическая спираль
11	10.449	4.138	0.287	-0.114	4.138	-1.188	4.305	4.305	0.032	Гиперболическая спираль
12	31.751	9.219	6.000	1.000	9.219	31.751	33.062	17.724	27.910	
13	3.765	3.509	0.188	0.175	3.509	0.661	3.570	3.323	1.306	Архимедова спираль
14	-10.000	15.000	1.047	-0.289	15.000	2.887	15.275	5.000	14.434	Строфоида
15	12.600	-8.700	1.159	-0.218	-8.700	-2.750	9.124	-0.960	-9.073	Циссоида
16	3.143	1.653	0.500	0.125	1.653	0.393	1.699	1.262	1.137	Циссоида
17	3.463	-0.341	0.750	0.250	-0.341	0.866	0.931	-0.840	0.401	Кардиоида
18	5.385	-0.769	2.043	0.292	-0.769	1.571	1.750	-1.050	-1.400	Гиперболическая спираль
19	22.255	2.226	2.226	0.223	2.226	4.953	5.430	-5.284	-1.251	Архимедова спираль
20	7.364	-2.678	0.841	-0.447	-2.678	-3.293	4.244	0.669	-4.191	Гипербола
21	8.105	-0.853	0.541	-0.277	-0.853	-2.248	2.404	0.427	-2.366	Гипербола
22	33.077	12.801	0.333	-0.129	12.801	-4.257	13.490	13.489	0.155	Гиперболическая спираль
23	3.733	-1.413	0.982	-0.134	-1.413	-0.499	1.499	-0.370	-1.452	Циссоида
24	10.427	-1.043	1.822	0.182	-1.043	1.900	2.167	-1.581	-1.482	Гиперболическая спираль
25	3.536	-1.741	0.963	-0.174	-1.741	-0.615	1.847	-0.490	-1.781	Циссоида
26	0.540	0.812	0.680	-0.177	0.812	-0.095	0.817	0.691	0.436	Строфоида
27	20.000	-2.857	1.159	-0.218	-2.857	-4.364	5.216	2.857	-4.364	Парабола
28	8.500	2.250	1.047	-0.289	2.250	-2.454	3.329	3.250	0.722	Улитка Паскаля
29	8.273	-0.540	0.857	0.143	-0.540	1.182	1.299	-1.247	0.365	Кардиоида
30	21.111	-1.235	0.451	-0.229	-1.235	-4.843	4.998	1.000	-4.897	Конхоида Никомеда
31	16.615	-1.022	0.982	-0.134	-1.022	-2.220	2.444	1.278	-2.084	Эллипс
32	7.738	0.924	0.857	0.143	0.924	1.105	1.441	-0.231	1.422	Гипербола
33	-8.641	2.542	0.545	0.091	2.542	-0.786	2.660	2.581	0.647	Строфоида
34	19.125	-1.266	0.476	-0.243	-1.266	-4.638	4.808	1.000	-4.703	Конхоида Никомеда

N_0	$\ddot{\rho}$	$\ddot{\varphi}$	W_ρ	W_φ	a	W_x	W_y	$ W_\tau $	W_n
1	0.067	0.000	-0.023	0.072	0.076	-0.067	0.036	0.059	0.048
2	0.190	0.000	-0.423	0.231	0.482	-0.475	-0.080	0.148	0.458
3	0.537	0.000	0.252	0.432	0.500	-0.134	0.482	0.497	0.060
4	-0.656	-0.021	-0.601	-0.717	0.936	0.245	-0.903	-0.698	0.622
5	-20.000	-0.009	-19.617	-4.331	20.089	0.320	-20.086	-20.069	0.906
6	0.336	-0.012	-0.084	0.332	0.342	-0.336	0.066	-0.236	0.248
7	0.359	0.022	0.061	1.671	1.672	-0.577	1.569	0.662	1.535
8	70.000	-0.009	68.600	14.574	70.131	-0.560	70.129	-70.112	1.620
9	0.188	0.023	0.047	-0.188	0.193	0.183	-0.062	-0.166	0.099
10	0.117	0.008	0.039	-0.117	0.124	0.120	-0.030	-0.110	0.055
11	2.788	0.013	2.653	-0.801	2.771	2.771	-0.016	2.771	0.037
12	0.661	0.000	-31.090	18.439	36.146	-24.699	26.391	9.038	34.998
13	1.256	0.063	1.140	1.468	1.858	0.845	1.655	1.392	1.231
14	-10.000	-0.048	-9.167	-8.179	12.285	2.500	-12.028	-10.547	6.299
15	7.500	-0.021	6.900	3.535	7.753	-0.480	7.738	-7.645	1.291
16	0.506	0.000	0.457	0.413	0.616	0.203	0.582	0.540	0.296
17	-0.091	0.000	-0.308	-0.170	0.352	-0.109	-0.335	-0.046	0.349
18	0.110	0.042	-0.349	-0.224	0.415	0.358	-0.209	-0.048	0.412
19	0.223	0.022	-0.880	1.486	1.727	-0.643	-1.603	0.995	1.411
20	1.947	-0.179	0.475	1.078	1.178	-0.487	1.072	-1.136	0.312
21	0.180	-0.128	-0.444	-0.564	0.718	-0.090	-0.712	0.685	0.215
22	8.665	0.012	8.118	-2.882	8.614	8.614	-0.074	8.612	0.173
23	0.432	-0.012	0.365	0.333	0.494	-0.074	0.489	-0.455	0.193
24	0.104	0.018	-0.242	-0.190	0.308	0.244	-0.187	-0.050	0.304
25	0.656	-0.021	0.549	0.532	0.764	-0.122	0.754	-0.695	0.318
26	-0.105	-0.039	-0.122	-0.308	0.331	0.099	-0.316	-0.085	0.320
27	0.816	-0.021	-0.136	0.831	0.842	-0.816	0.208	-0.621	0.569
28	0.000	-0.048	-0.708	-1.708	1.849	1.125	-1.467	0.780	1.676
29	-0.067	0.000	-0.236	-0.154	0.282	-0.038	-0.279	-0.042	0.278
30	0.274	-0.109	-0.837	-1.728	1.920	-0.000	-1.920	1.881	0.384
31	0.126	-0.012	-0.171	0.075	0.187	-0.157	-0.100	0.003	0.187
32	0.335	0.000	0.177	0.264	0.318	-0.084	0.307	0.316	0.033
33	0.495	0.000	0.566	0.462	0.731	0.244	0.689	0.404	0.609
34	0.316	-0.114	-0.809	-1.569	1.765	0.000	-1.765	1.726	0.367